

# Worldsheet

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## Abstract

We consider the noncommutativity parameter of the space-time as a bosonic worldsheet field. By finding a fermionic super-partner for it, we can find star products between the boson-boson, boson-fermion and fermion-fermion fields of superstring worldsheet and also between superfields of the worldsheet superspace. We find a two dimensional action for the noncommutativity parameter and its fermionic partner. We discuss the symmetries of this action.

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# 1 Introduction

The study of open strings in the presence of background fields enables us to explain the noncommutativity on the D-brane worldvolume [1, 2]. In the most of these attempts the noncommutativity parameter is constant i.e., it is independent of the spacetime coordinates. It is given in terms of a constant background NS $\otimes$ NS  $B$ -field and a constant closed string metric  $g_{\mu\nu}$  [1]. The noncommutativity parameter also can be non-constant [3, 4, 5]. One can achieve to this, by introducing a nontrivial  $B$ -field [4] or more general, a curved background metric  $g_{\mu\nu}$  with nontrivial  $B$ -field [5].

The fact that the noncommutativity parameter is not constant implies that, indirectly this parameter depends on the coordinates of the string worldsheet. We consider the noncommutativity parameter  $\Theta^{\mu\nu}$  as a bosonic field of the string worldsheet. Worldsheet supersymmetry enables us to find a fermionic super-partner for it. We call this fermionic field  $\omega^{\mu\nu}$ . We study supersymmetry of the  $\Theta\omega$ -system. That is, we find the supersymmetry transformations of the fields  $\Theta^{\mu\nu}$  and  $\omega^{\mu\nu}$ . We show that the algebra of these transformations is closed.

Imposing the worldsheet supersymmetry to the noncommutativity relation of the space-time, creates star products between the boson-boson, boson-fermion and fermion-fermion fields. By these products we can find the noncommutativity of the superfields associated to the worldsheet superspace. Therefore, this noncommutativity is compatible with supersymmetry. Furthermore, we shall show that star product of two superfields is a superfield. In other words, noncommutativity parameter of the superfield coordinates is a superfield.

Some transformation properties of the components of the supersymmetrized local noncommutativity parameter will be discussed. By quantizing the system, the algebra of the supercurrent is studied. In addition to the supersymmetry, the action of the  $\Theta\omega$ -system also is symmetric under the reparametrization of the worldsheet coordinates, the Weyl scaling of the worldsheet metric, the transformations that are induced by Poincaré transformations and another linear transformation of  $\Theta^{\mu\nu}$  and  $\omega^{\mu\nu}$ . We study the two later symmetries in detail.

This paper is organized as follows. In section 2, supersymmetry of the noncommutativity parameter and its fermionic partner is discussed. In section 3, we find star products between the worldsheet bosons and fermions and also between superfields of worldsheet superspace. In section 4, the action of these new fields, the quantization of these fields, the supercurrent and its algebra will be obtained. In section 5, we study the induced Poincaré symmetry and another symmetry of this system.

## 2 Supersymmetry transformations of $\Theta^{\mu\nu}$ and $\omega^{\mu\nu}$

We know that for a noncommutative space with the coordinates  $\{X^\mu\}$  and the noncommutativity parameter  $\Theta^{\mu\nu}$  there is

$$X^\mu * X^\nu = X^\mu X^\nu + \frac{i}{2} \Theta^{\mu\nu} , \quad (1)$$

which leads to the commutator

$$[X^\mu, X^\nu]_* = i \Theta^{\mu\nu} . \quad (2)$$

Note that in string theory, the equation (1) is definition of the star product between two worldsheet bosons  $X^\mu$  and  $X^\nu$ . It is not definition of the parameter  $\Theta^{\mu\nu}$ . Later we shall find an appropriate action for  $\Theta^{\mu\nu}$ .

Now consider superstring theory with worldsheet supersymmetry. It has the action

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma (\partial_a X^\mu \partial^a X_\mu - i \bar{\psi}^\mu \rho^a \partial_a \psi_\mu) , \quad (3)$$

where  $\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$  is a Majorana spinor of worldsheet. The equations of motion extracted from this action are

$$\partial_+ \psi_-^\mu = \partial_- \psi_+^\mu = \partial_+ \partial_- X^\mu = 0 , \quad (4)$$

where  $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$ . Under the worldsheet supersymmetry transformations, this action is invariant. These transformations are

$$\begin{aligned} \delta X^\mu &= \bar{\epsilon} \psi^\mu , \\ \delta \psi^\mu &= -i \rho^a \partial_a X^\mu \epsilon , \end{aligned} \quad (5)$$

where  $\epsilon$  is a constant anti-commuting infinitesimal Majorana spinor. According to the equations of motion (4), supersymmetry means that  $\partial_\pm X^\mu$  appears like  $\psi_\pm^\mu$  and vice-versa.

From the equation (1) and the supersymmetry transformations (5), we have

$$\frac{i}{2} \delta \Theta^{\mu\nu} = \bar{\epsilon} (\psi^\mu * X^\nu + X^\mu * \psi^\nu - \psi^\mu X^\nu - X^\mu \psi^\nu) . \quad (6)$$

This transformation can be written as

$$\delta \Theta^{\mu\nu} = \bar{\epsilon} \omega^{\mu\nu} , \quad (7)$$

where  $\omega^{\mu\nu}$  is defined as

$$\frac{i}{2}\omega^{\mu\nu} = \psi^\mu * X^\nu + X^\mu * \psi^\nu - \psi^\mu X^\nu - X^\mu \psi^\nu , \quad (8)$$

or in the commutator form, it is

$$i\omega^{\mu\nu} = [\psi^\mu , X^\nu]_* - [\psi^\nu , X^\mu]_* . \quad (9)$$

Also the equations (5) and (8) give the supersymmetry transformation of  $\omega^{\mu\nu}$  i.e.,

$$\frac{i}{2}\delta\omega^{\mu\nu} = -i\rho^a\epsilon(\partial_a X^\mu * X^\nu + X^\mu * \partial_a X^\nu - \partial_a X^\mu X^\nu - X^\mu \partial_a X^\nu) . \quad (10)$$

By using the equation (1) this transformation takes the simple form

$$\delta\omega^{\mu\nu} = -i\rho^a\partial_a\Theta^{\mu\nu}\epsilon . \quad (11)$$

Note that the transformations of the equations (2) and (9) also lead to the results (7) and (11), respectively. Now we have a new Majorana spinor  $\omega^{\mu\nu} = \begin{pmatrix} \omega_-^{\mu\nu} \\ \omega_+^{\mu\nu} \end{pmatrix}$  which is antisymmetric under the exchange of the indices  $\mu$  and  $\nu$ .

The transformations (7) and (11) form a closed algebra, that is for two successive transformations  $\delta_\epsilon$  and  $\delta_{\epsilon'}$  we obtain

$$\begin{aligned} [\delta_\epsilon , \delta_{\epsilon'}]\Theta^{\mu\nu} &= 2i\bar{\epsilon}\rho^a\epsilon'\partial_a\Theta^{\mu\nu} , \\ [\delta_\epsilon , \delta_{\epsilon'}]\omega^{\mu\nu} &= 2i\bar{\epsilon}\rho^a\epsilon'\partial_a\omega^{\mu\nu} . \end{aligned} \quad (12)$$

Note that closeness of algebra means that the commutator of two supersymmetry transformations gives a spatial translation. This can be seen from the above equations.

### 3 Star product between various fields

Now we find star product between the worldsheet fields and between superfields. Making derivative of the equation (1) with respect to the light-cone coordinates  $\sigma^\pm$ , leads to the equation

$$\frac{i}{2}\partial_\pm\Theta^{\mu\nu} = \partial_\pm X^\mu * X^\nu + X^\mu * \partial_\pm X^\nu - \partial_\pm X^\mu X^\nu - X^\mu \partial_\pm X^\nu . \quad (13)$$

According to the supersymmetry this equation gives

$$\frac{i}{2}\Omega^{\mu\nu} = \psi^\mu * X^\nu + X^\mu * \psi^\nu - \psi^\mu X^\nu - X^\mu \psi^\nu , \quad (14)$$

where we introduced the spinor  $\Omega^{\mu\nu}$  as super-partner of  $\Theta^{\mu\nu}$ . The equations (8) and (14) give the equality  $\Omega^{\mu\nu} = \omega^{\mu\nu}$ . Therefore, the equations (7), (8) and (11) imply  $\Theta^{\mu\nu}$  and  $\omega^{\mu\nu}$  are super-partner of each other. Again note that the equation (8) is definition of star product between the worldsheet bosons  $\{X^\mu\}$  and the worldsheet fermions  $\{\psi^\mu\}$ . It is not definition of the spinor  $\omega^{\mu\nu}$ . We shall give an appropriate action for  $\omega^{\mu\nu}$ .

From the equation (13) we obtain

$$\partial_+ X^\mu * \partial_- X^\nu + \partial_- X^\mu * \partial_+ X^\nu = \partial_+ X^\mu \partial_- X^\nu + \partial_- X^\mu \partial_+ X^\nu + \frac{i}{2} \partial_+ \partial_- \Theta^{\mu\nu}, \quad (15)$$

Using the supersymmetry, leads to the equation

$$\psi_+^\mu * \psi_-^\nu + \psi_-^\mu * \psi_+^\nu = \psi_+^\mu \psi_-^\nu + \psi_-^\mu \psi_+^\nu + \frac{i}{2} \partial_+ \partial_- \Theta^{\mu\nu}. \quad (16)$$

This shows the star product between the components of the spinor fields of the string worldsheet. This product, similar to the equation (8), naturally is defined by two terms that contain star product. The equation (16) also can be obtained from the equation (8).

According to the equations (1), (8) and (16) we can find star product and noncommutativity parameter for a space with worldsheet superfields as its coordinates. A superfield in general has the form

$$Y^\mu = X^\mu + \bar{\theta} \psi^\mu + \frac{1}{2} \bar{\theta} \theta B^\mu, \quad (17)$$

where the Grassmann coordinates  $\theta^1$  and  $\theta^2$  form a two dimensional Majorana spinor  $\theta = \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix}$ . Under the supersymmetry, this field transforms as  $\delta Y^\mu = \bar{\epsilon} Q(Y^\mu)$ , where the generator  $Q = \frac{\partial}{\partial \theta} + i \rho^a \theta \partial_a$  represents supersymmetry on the superspace. The commutator of two superfields with star product is

$$[Y^\mu, Y^\nu]_* = i \Phi^{\mu\nu} \equiv i(\Theta^{\mu\nu} + \bar{\theta} \omega^{\mu\nu} + \frac{1}{2} \bar{\theta} \theta \Gamma^{\mu\nu}), \quad (18)$$

where the matrix  $\Gamma^{\mu\nu}$  is

$$\Gamma^{\mu\nu} = i(\bar{\psi}^\mu * \psi^\nu - \bar{\psi}^\nu * \psi^\mu). \quad (19)$$

The antisymmetric tensor  $\Phi^{\mu\nu}$  shows the noncommutativity parameter of the space  $\{Y^\mu\}$ . This parameter depends on the superspace coordinates  $\sigma, \tau, \theta^1$  and  $\theta^2$ . The various components of  $\Phi^{\mu\nu}$  i.e.,  $\Theta^{\mu\nu}$ ,  $\omega^{\mu\nu}$  and  $\Gamma^{\mu\nu}$  show the noncommutativity of  $X - X$ ,  $X - \psi$  and  $\psi - \psi$  subspaces.

We know that usual product of two superfields  $Y^\mu$  and  $Y^\nu$  is a superfield i.e.,  $\delta(Y^\mu Y^\nu) = \bar{\epsilon}Q(Y^\mu Y^\nu)$ . This also holds by star product i.e.,  $Y^\mu * Y^\nu$  is a superfield. Now we show this. The star product is

$$Y^\mu * Y^\nu = Y^\mu Y^\nu + \frac{i}{2}\Lambda^{\mu\nu} , \quad (20)$$

where  $\Lambda^{\mu\nu}$  is

$$\begin{aligned} \Lambda^{\mu\nu} &= \Theta^{\mu\nu} + \bar{\theta}\omega^{\mu\nu} + \frac{1}{2}\bar{\theta}\theta\lambda^{\mu\nu} , \\ \lambda^{\mu\nu} &= 2i(\bar{\psi}^\mu * \psi^\nu - \bar{\psi}^\nu * \psi^\mu) . \end{aligned} \quad (21)$$

Since  $Y^\mu Y^\nu$  is a superfield, it is sufficient to show that  $\Lambda^{\mu\nu}$  also is a superfield. Using the equations (1) and (8) for  $\Theta^{\mu\nu}$  and  $\omega^{\mu\nu}$  and also applying the geometrical transformations  $\delta\theta = \epsilon$  and  $\delta\sigma^a = i\bar{\epsilon}\rho^a\theta$ , lead to the equation

$$\delta\Lambda^{\mu\nu} = \bar{\epsilon}Q(\Lambda^{\mu\nu}) , \quad (22)$$

that is,  $\Lambda^{\mu\nu}$  is a superfield and therefore,  $Y^\mu * Y^\nu$  is superfield,

$$\delta(Y^\mu * Y^\nu) = \bar{\epsilon}Q(Y^\mu * Y^\nu) . \quad (23)$$

In the same way, it is easy to show that the noncommutativity parameter  $\Phi^{\mu\nu}$  also is superfield.

The superfield  $\Lambda^{\mu\nu}$  gives the following supersymmetry transformations

$$\begin{aligned} \delta\Theta^{\mu\nu} &= \bar{\epsilon}\omega^{\mu\nu} , \\ \delta\omega^{\mu\nu} &= -i\rho^a\epsilon\partial_a\Theta^{\mu\nu} + \epsilon\lambda^{\mu\nu} , \\ \delta\lambda^{\mu\nu} &= -i\bar{\epsilon}\rho^a\partial_a\omega^{\mu\nu} . \end{aligned} \quad (24)$$

For compatibility with the equations (7) and (11),  $\lambda^{\mu\nu}$  and  $\delta\lambda^{\mu\nu}$  should vanish i.e.,

$$\bar{\psi}^\mu * \psi^\nu = \bar{\psi}^\mu \psi^\nu , \quad (25)$$

$$\rho^a\partial_a\omega^{\mu\nu} = 0 . \quad (26)$$

Since  $\Theta^{\mu\nu}$  is super-partner of  $\omega^{\mu\nu}$ , the equation (26) implies

$$\partial^a\partial_a\Theta^{\mu\nu} = 0 . \quad (27)$$

The equations (16), (25) and (27) tell us that star product between components of the worldsheet fermions  $\{\psi^\mu\}$  is usual product that is,  $\psi_A^\mu * \psi_B^\nu = \psi_A^\mu \psi_B^\nu$ , where  $A, B \in \{+, -\}$ .

## 4 Action of the fields $\Theta^{\mu\nu}$ and $\omega^{\mu\nu}$

According to the supersymmetry transformations (7) and (11) and the equations (26) and (27) we can write the following action for  $\Theta^{\mu\nu}$  and  $\omega^{\mu\nu}$ ,

$$S' = -\frac{1}{4\pi\alpha'} \int d^2\sigma (\partial_a \Theta^{\mu\nu} \partial^a \Theta_{\mu\nu} - i\bar{\omega}^{\mu\nu} \rho^a \partial_a \omega_{\mu\nu}) . \quad (28)$$

The equations (26) and (27) are equations of motion that can be extracted from this action. The fields  $\Theta^{\mu\nu}(\sigma, \tau)$  and  $\omega^{\mu\nu}(\sigma, \tau)$  are massless bosons and fermions that live in the worldsheet of superstring.

Under the supersymmetry transformations (7) and (11) this action is invariant. The supercurrent associated to this symmetry is

$$\mathcal{J}_a = \frac{1}{2} \rho^b \rho_a \omega^{\mu\nu} \partial_b \Theta_{\mu\nu} . \quad (29)$$

This is a conserved current i.e.,  $\partial^a \mathcal{J}_a = 0$ . The light-cone components of this current are

$$\begin{aligned} \mathcal{J}_+ &= \omega_+^{\mu\nu} \partial_+ \Theta_{\mu\nu} , \\ \mathcal{J}_- &= \omega_-^{\mu\nu} \partial_- \Theta_{\mu\nu} . \end{aligned} \quad (30)$$

Now we verify the quantization of the  $\Theta\omega$ -system. Quantization of the fermionic degrees of freedom is achieved by imposing the canonical anti-commutation relations. The canonical momenta conjugate to  $\omega_{\pm}^{\mu\nu}$  are

$$\Pi_{\pm}^{\mu\nu}(\sigma, \tau) = -\frac{i}{4\pi\alpha'} \omega_{\pm}^{\mu\nu} . \quad (31)$$

Using the equal  $\tau$  anti-commutators, we obtain

$$\{\omega_A^{\mu\nu}(\sigma, \tau) , \omega_B^{\rho\lambda}(\sigma', \tau)\} = \frac{\pi}{2} (\eta^{\mu\rho} \eta^{\nu\lambda} - \eta^{\mu\lambda} \eta^{\nu\rho}) \delta(\sigma - \sigma') \delta_{AB} . \quad (32)$$

Both sides under the exchanges  $\mu \leftrightarrow \nu$  or  $\rho \leftrightarrow \lambda$  change their signs. Also both sides under the exchanges  $\mu \leftrightarrow \rho$ ,  $\nu \leftrightarrow \lambda$ ,  $\sigma \leftrightarrow \sigma'$  and  $A \leftrightarrow B$  are invariant. The canonical momentum conjugate to  $\Theta^{\mu\nu}$  is

$$\Pi^{\mu\nu}(\sigma, \tau) = \frac{1}{2\pi\alpha'} \partial_{\tau} \Theta^{\mu\nu} . \quad (33)$$

Quantization of the bosonic degrees of freedom can be obtained by canonical commutation relations

$$[\partial_{\pm} \Theta^{\mu\nu}(\sigma, \tau) , \partial_{\pm} \Theta^{\rho\lambda}(\sigma', \tau)] = \pm \frac{i\pi}{4} (\eta^{\mu\rho} \eta^{\nu\lambda} - \eta^{\mu\lambda} \eta^{\nu\rho}) \delta'(\sigma - \sigma') , \quad (34)$$

$$[\partial_+ \Theta^{\mu\nu}(\sigma, \tau), \partial_- \Theta^{\rho\lambda}(\sigma', \tau)] = 0. \quad (35)$$

Under the exchanges  $\mu \leftrightarrow \rho$ ,  $\nu \leftrightarrow \lambda$  and  $\sigma \leftrightarrow \sigma'$  these equations are invariant.

According to the equations (32), (34) and (35) the algebra that the supercurrent (30) generates, is

$$\begin{aligned} \{\mathcal{J}_+(\sigma), \mathcal{J}_+(\sigma')\} &= \pi\delta(\sigma - \sigma')\mathcal{T}_{++}(\sigma), \\ \{\mathcal{J}_-(\sigma), \mathcal{J}_-(\sigma')\} &= \pi\delta(\sigma - \sigma')\mathcal{T}_{--}(\sigma), \\ \{\mathcal{J}_+(\sigma), \mathcal{J}_-(\sigma')\} &= 0, \end{aligned} \quad (36)$$

where  $\mathcal{T}_{++}$  and  $\mathcal{T}_{--}$  are the non-zero light-cone components of the energy momentum tensor, extracted from the action (28),

$$\begin{aligned} \mathcal{T}_{++} &= \partial_+ \Theta^{\mu\nu} \partial_+ \Theta_{\mu\nu} + \frac{i}{2} \omega_+^{\mu\nu} \partial_+ \omega_{+\mu\nu}, \\ \mathcal{T}_{--} &= \partial_- \Theta^{\mu\nu} \partial_- \Theta_{\mu\nu} + \frac{i}{2} \omega_-^{\mu\nu} \partial_- \omega_{-\mu\nu}. \end{aligned} \quad (37)$$

The algebra (36) is similar to the algebra of the supercurrent associated with the supersymmetry transformations (5).

## 5 Other symmetries

The superstring action (3) under the Poincaré transformations is invariant. These transformations are

$$\begin{aligned} \delta X^\mu &= a^\mu{}_\nu X^\nu + b^\mu, \\ \delta \psi^\mu &= a^\mu{}_\nu \psi^\nu, \end{aligned} \quad (38)$$

where  $a_{\mu\nu}$  is a constant antisymmetric matrix and  $b^\mu$  is a constant vector that shows translation.

Application of the transformations (38) in the equations (1) and (8) induces the following transformations to the fields  $\Theta^{\mu\nu}$  and  $\omega^{\mu\nu}$ ,

$$\delta \Theta^{\mu\nu} = (a^\mu{}_\rho \delta^\nu{}_\lambda - a^\nu{}_\rho \delta^\mu{}_\lambda) \Theta^{\rho\lambda}, \quad (39)$$

$$\delta \omega^{\mu\nu} = (a^\mu{}_\rho \delta^\nu{}_\lambda - a^\nu{}_\rho \delta^\mu{}_\lambda) \omega^{\rho\lambda}. \quad (40)$$

Since the translation  $b^\mu$  is independent of the spacetime coordinates  $\{X^\mu\}$ , it does not appear to these transformations. Note that application of the transformations (38) in the equations (2) and (9) also gives the results (39) and (40).



The action (28) under the transformations (39) and (40) is invariant. This invariance leads to the current

$$J_a^{\mu\nu\rho\lambda} = \frac{1}{\pi}(\Theta^{\mu\nu}\partial_a\Theta^{\rho\lambda} - \Theta^{\rho\lambda}\partial_a\Theta^{\mu\nu} + i\bar{\omega}^{\mu\nu}\rho_a\omega^{\rho\lambda}) . \quad (41)$$

Under the exchange of the indices, this current satisfies the following identities

$$J_a^{\mu\nu\rho\lambda} = -J_a^{\nu\mu\rho\lambda} = -J_a^{\mu\nu\lambda\rho} = -J_a^{\rho\lambda\mu\nu} . \quad (42)$$

According to the equations of motion, this is a conserved current i.e.,

$$\partial^a J_a^{\mu\nu\rho\lambda} = 0 . \quad (43)$$

Since the transformation (39) only rotates  $\Theta^{\mu\nu}$  but does not translate it, there is no current for translation.

The action (28) also is symmetric under the following linear transformations

$$\begin{aligned} \delta\Theta^{\mu\nu} &= A^{\mu\nu}{}_{\rho\lambda}\Theta^{\rho\lambda} + b^{\mu\nu} , \\ \delta\omega^{\mu\nu} &= A^{\mu\nu}{}_{\rho\lambda}\omega^{\rho\lambda} , \end{aligned} \quad (44)$$

where  $b^{\mu\nu}$  and  $A^{\mu\nu}{}_{\rho\lambda}$  satisfy the following identities

$$\begin{aligned} A^{\mu\nu}{}_{\rho\lambda} &= -A^{\nu\mu}{}_{\rho\lambda} = -A^{\mu\nu}{}_{\lambda\rho} = -A_{\rho\lambda}{}^{\mu\nu} , \\ b^{\mu\nu} &= -b^{\nu\mu} . \end{aligned} \quad (45)$$

The currents associated to the transformations (44), are the current (41) and the current

$$P_a^{\mu\nu} = \frac{1}{\pi}\partial_a\Theta^{\mu\nu} . \quad (46)$$

This current is a result of the translation of  $\Theta^{\mu\nu}$ . Also it is a conserved current i.e.,

$$\partial^a P_a^{\mu\nu} = 0 . \quad (47)$$

In addition to the supersymmetry, the induced Poincaré symmetry and the symmetry under the transformations (44), the action (28) similar to the action (3), also is symmetric under the reparametrization of the parameters  $\sigma$  and  $\tau$  and the Weyl scaling of the worldsheet metric.

## 6 Conclusions

We considered the noncommutativity parameter as a bosonic field of the string worldsheet. By applying the worldsheet supersymmetry, a super-partner was associated to it. We found the supersymmetry transformations of these new fields. Therefore, in a natural way we obtained star products between the boson-boson, boson-fermion and fermion-fermion fields. According to these products the noncommutativity parameter of the superfields (functions on the worldsheet superspace) was obtained. We saw that star product of two superfields (like usual product of them) is a superfield. Therefore, the noncommutativity parameter of these superfields transforms as a superfield.

From the equations of motion and the supersymmetry transformations of this system, we obtained action of the system. By quantizing the system, we obtained the algebra of the supercurrent. For this action we extracted the induced Poincaré symmetry and the current associated to it. This four indices current satisfies some identities. We showed that invariance under the transformations (44), leads to the above four indices current and a conserved current for the translation of the noncommutativity parameter. Also this action under the reparametrization of the worldsheet coordinates, the Weyl scaling of the worldsheet metric is invariant.

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